Continuing a Coherent Mathematics Program

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A Study Document for Educators at the Junior High Level



Continuing a Coherent Mathematics Program

A Study Document for Educators at the Junior High Level This document is available on the Internet at **<PLANS.EDnet.ns.ca>**. For further information about *Continuing a Coherent Mathematics Program: A Study Document for Educators at the Junior High Level,* please contact

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Continuing a Coherent Mathematics Program: A Study Document for Educators at the Junior High Level

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Introduction

Continuing a Coherent Mathematics Program: A Study Document for Educators at the Junior High Level contains an analysis of the results of the Junior High Mathematics Program Assessment administered in 2004. It is a companion document to *Toward a Coherent Mathematics Program: A Study Document for Educators* (2002), which addressed the results of the Elementary Mathematics Program Assessment. Together these documents give Nova Scotia educators information to further improve mathematics teaching and learning in our schools.

This report contains an analysis of student responses to the questions in the program assessment. It also provides examples of student work and the rubrics used to score them. Specifically, it addresses what we have learned from these responses. As well, it gives indicators on particular strands of the curriculum that may require increased focus and attention.

Educators are encouraged to read this document, discuss the contents, and reflect on how the suggestions can be applied.

Background

Did you know ...

Almost 80 percent of grade 8 teachers either agree or strongly agree that provincial assessments are worthwhile. In the spring of 2004, 11 787 grade 8 public school students participated in the Junior High Mathematics Program Assessment. French immersion students wrote a French translation of the English version. Some participating students were permitted certain documented adaptations in order to do the assessment, but these adaptations in no way modified the assessment content or requirements. Students who followed an approved alternative curriculum as described in their individual program plans were excluded from the assessment. Students enrolled in the Conseil scolaire acadien provincial did not write the assessment, as the curriculum for these students did not match the content of the assessment.

The assessment was designed to address the outcomes articulated in the Atlantic Canada Mathematics curriculum for grades 6, 7, and 8. Assessment questions covered all the major curriculum components and were distributed over three levels of complexity. For computational tasks, students were required to respond to questions in mental math, estimation, and paper-and-pencil procedures. The questions were timed, and calculators were not permitted. There were 30 selected-response (multiple choice) questions, 10 short-response questions, and 5 extended-response questions for the tasks involving concepts and applications. For these questions, students were encouraged to use concrete materials, rulers, and calculators.

Following the administration of the assessment, a random sample of student booklets was collected for central marking. In total, 4491 student booklets were marked in Halifax by teachers from across the province representing the regional school boards. The results of the assessment were published in the 2004 Minister's Report to Parents and also in a more comprehensive report that was distributed to schools. These reports are available on the Program of Learning Assessment for Nova Scotia (PLANS) website at plans.EDnet.ns.ca>.

Within the assessment, questions were distributed according to the following guidelines:

- approximately 25 percent of questions assessed knowledge and procedures
- approximately 50 percent of questions assessed comprehension and routine application
- approximately 25 percent of questions assessed non-routine application and problem solving

During the in-service sessions that preceded the administration of the assessment, all grade 8 teachers were asked to complete a questionnaire. The questionnaire was designed to collect the opinions of teachers as they related to assessment, the math curriculum, and strategies for teaching and learning. The information gathered from these questionnaires will be shared with the relevant stakeholders in education. Highlights of teachers' responses have been included in this report.

Coherent Programming

Did you know ..

More than 90 percent of teachers agree or strongly agree that there is a need for professional development concerning assessment. The Atlantic Canada Mathematics curriculum was carefully conceived to emphasize a logical, developmental sequence of mathematics from grade to grade to the end of the public school program. Thus, knowing the curriculum and how it fits in that continuum is a critical component and a first step in the process of ensuring a coherent program. The curriculum and its associated print resources specify what students are expected to know and be able to do at each grade level in order for them to be well prepared for the next grade. Curriculum documents contain necessary information and helpful advice for teachers to create meaningful learning opportunities for their students.

The following suggestions taken from the elementary document, *Toward a Coherent Mathematics Program: A Study Document for Educators*, continue to be relevant in the junior high context.

What can be done to provide students with a coherent program?

1. Teachers at all grade levels need to plan and organize their mathematics program around the outcomes assigned to their specific grade(s), selecting text and other resources to match these outcomes.

Vigilance is required to keep the mathematical activities focussed on the prescribed learning outcomes and focussed on what students need to be doing in order to grow in their understanding and abilities with regard to these outcomes.

- 2. Teachers from all grade levels within a school should work co-operatively to plan the mathematics program. Each grade level is a part of a continuum of mathematics instruction and learning. Thus, each teacher's role is a critical link in the process of ensuring that each student has the requisite mathematical knowledge and skills for each successive grade level.
- 3. If there are teachers in schools with particular interest and expertise in mathematics education, then opportunities could be provided for these teachers to support and share their content knowledge and understanding of mathematics with colleagues that are less comfortable and confident in this subject area.

Teachers must not hesitate to seek support in areas of the mathematics program that are less familiar to them. Since teaching for understanding is a major goal of the Atlantic Canada curriculum, it is important that teachers have that understanding themselves in order to structure lessons and create learning opportunities for their students to gain clarity. School-based mathematics leaders and math mentors as well as regional math leaders are available in all boards to help teachers implement the program as it is written with the pedagogy that it implies.

- 4. Teachers at each grade level should endorse in their practices what it means to do mathematics. Students, for their part, must recognize that mathematics is about thinking and logical reasoning, looking for patterns and relationships, solving problems, and clearly communicating ideas and strategies. Such recognition can be achieved when students engage in these activities on a regular basis. If the students' classroom experience is one of listening to ideas being explained by the teacher, copying set procedures, and rehearsing a rule by applying it again and again to similar questions, then they will acquire a view of what it means to do mathematics that is very different from what is articulated by the Atlantic Canada mathematics curriculum. Students gain a thorough understanding of a concept if they recognize exemplars and non-exemplars of that concept in any of its representations (contexts, symbols, concrete models, pictorial models); if they know and use the language associated with that concept; if they can go from one representation of the concept to another; if they make connections to other concepts; and if they can apply the concept in new and novel situations. Students must be encouraged to think deeply, be willing to struggle with challenging tasks, persevere, take risks, and have a productive disposition.
- 5. In order to accurately gauge the progress of students, classroom assessment practices must reflect the curriculum emphases of understanding concepts and procedures and of solving problems. Whether the assessment is through portfolios, projects, observations, interviews, or paper-and-pencil instruments, the tasks and questions posed should require responses that give teachers insight on individual student's growth and development with respect to understanding and problem solving.

Every opportunity should be taken by the teacher to integrate previously learned concepts with concepts currently being assessed. The goal is to provide students with a coherent program that will prepare them for the next grade where they will continue to accumulate and consolidate their knowledge and their understanding.

6. All teachers of mathematics require adequate and appropriate resources, both print and concrete, to deliver a coherent program. Teachers need ongoing professional development opportunities to increase their personal understanding of mathematics and to broaden their teaching methodologies. They need time to learn, reflect, and implement existing and new curriculum documents and resources as well as opportunities to expand their classroom assessment standards and practices. All stakeholders are obligated to explore ways to attend to these needs.

Teachers have the central role in providing students with a coherent mathematics program. Implementation of these six recommendations will require the support of all teachers assigned to teach mathematics. In turn, teachers need the understanding and support of their school boards and administrators in their quest to improve the teaching and learning of mathematics in accordance with the Atlantic Canada mathematics curriculum.

Did you know

More than 80 percent of grade 8 teachers reported that provincial assessments have a positive impact on curriculum delivery.

How to Use This Study Document

This document is organized around four major content areas as well as a section devoted to how students did on a variety of computational tasks. The content areas are

- Number Concept and Operations
- Patterns and Relationships
- Measurement and Geometry
- Data Management and Probability

Assessment questions are presented within each content area, along with comments about the quality of student responses and what we have learned from their responses. Strategies and suggestions are included to help educators to improve student learning.

Teachers

- 1. Use this study document along with the curriculum guide to plan for instruction and student learning.
- 2. While planning for a unit of work on a specific strand, read through the corresponding sections of this document. Consider the comments and suggestions that are provided. Reflect on your own teaching practices and what you know about the students in your class. Consider the common misconceptions and difficulties that are presented so you can anticipate where your student may struggle and plan how to avoid these potential problems.
- 3. Select questions from the Junior High Mathematics Program Assessment and administer these to your students. Compare the results you obtain with the results reported provincially. Check to see if your students have similar misconceptions or experience difficulties like those reported on in this document. This may be conducted as a pre-test and then again as a post-test. Use the suggestions, comments, and examples to remediate where necessary.
- 4. Remember that you play an integral part within a learning continuum of mathematics education that begins in elementary school and continues on through high school. Reflect on your contribution to the delivery of a coherent mathematics program to students at your school. This should help you set personal professional goals and learning targets for your students.

School Staff

- 1. All the mathematics teachers within a school should work toward the goal of providing students with a coherent mathematics program. To this end, a whole-school approach is required. Arrangements should be made to meet and consider the comments and suggestions in this document.
- 2. The initial meeting should have the expressed purpose of addressing the six recommendations that follow the question, What can be done to provide students with a coherent program?
- 3. Subsequent meetings should address the collective role of mathematics teachers in relation to the comments, suggestions, and student work that are presented in each of the sections in this document.
- 4. The information in this document may also provide a basis for a professional learning community as school staff focus on their professional development needs.

School Boards

- 1. Board curriculum supervisors/consultants, mathematics committees/leadership teams, math mentors, and school-based leaders should consider how they can provide help and support to individual teachers, groups of teachers, and school staffs as they work toward the major goal of providing students with a coherent mathematics program.
- 2. Comments and suggestions found in this document are an agenda for action and an excellent source of material for workshops.

Did you know ..

Half of the teachers responded that they did not expect their students to do well on the provincial assessment.

Computational Tasks (GCO B)

The questions in this section assessed students' knowledge of number facts, mental math strategies, estimation and rounding skills, and paper-and-pencil procedures. For the most part, the questions required students to find or recognize correct numerical answers to computations.

Mental math, estimation, and rounding are important mathematical skills needed to solve problems in real-world scenarios and are important facets of the mathematics program. Mental math and estimation require a more flexible way of thinking and necessitate learning and practising specific strategies. Many of the curriculum outcomes in GCO (B), at each grade level, "refer to fact learning, mental math, and/or estimation strategies," but in a broader sense, there are outcomes in most of the GCOs that would have direct or indirect mental math and estimation possibilities.

The graph below shows the provincial mean percentage score for computational tasks as 33.2 percent. It also shows the school board mean percentage scores ranging from 29.7 to 37.1.



Mental Mathematics

What we learned from student responses

Students were asked 20 questions to be solved mentally. The questions were timed so that students did not have the time to work out the computation using paper and pencil. No calculators were to be used. The questions assessed the concepts and procedures in GCO (B) of the Atlantic Canada Mathematics Curriculum.

1. (-9)+(-12)+8 (Grade 7/SCO B11)

Approximately 45 percent of grade 8 students correctly answered -13. Many responded with 29 or -29, indicating that they ignored the signs or misread them. Students should have had many opportunities to develop an understanding of adding integers using context, two-colour counters, and number lines. They should be able to recall addition facts confidently and quickly.

2. 23 000-4000 (GRADE 6/SCO B11)

Grade 8 students responded well, with 75 percent correctly answering 19 000.

3. 825+651+175 (Grade 6/SCO B11)

In this case, 53 percent of the students correctly responded 1651. A common answer was 1551. A number of students appeared to have insufficient time to solve the problem. Recognizing the compatible pair of 825 and 175 will enable them to do this quickly and efficiently.

4. $\frac{2}{3} + \frac{5}{6}$ (GRADE 8/SCO B6)

Only 45 percent answered $\frac{9}{6}$ or $\frac{3}{2}$ or $1\frac{1}{2}$, which were all acceptable answers. Common answers given were $\frac{7}{9}$ and $\frac{7}{18}$. Visualizing fractions by using models such as those found in the Fraction Factory will enable students to be more successful at these types of tasks.

725+150+275=? hmmm ... 725+275=1000 Therefore, the answer is 1150.

5. $\frac{1}{4} \times 284$ (Grade 7/SCO B7)

Many students did not answer this question and very few correctly answered 71. Students need practice in multiplying and dividing by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and recognizing patterns. A number of errors observed seem to be directly related to incorrect number facts. Perhaps verbalizing this as "one quarter …" might be helpful.

6. 5.6-1.8 (GRADE 6/SCO B11)

Results indicated that 45 percent of students correctly responded 3.8. A number of students answered 4.2, indicating that they subtracted the smaller digit from the larger one regardless of its position. A suggested strategy would be to change 1.8 to 2 and 5.6 to 5.8 by adding 0.2 to both. The question becomes 5.8 – 2.0. This avoids the need for regrouping.

7. 0.01×1200 (GRADE 6/SCO B12)

As well, 45 percent of grade 8 students correctly answered 12. Students wrote answers such as 0.0012, 1200.01, or 12.01. Even though the question was read to the students as "What is one-hundredth of twelve hundred?" many students did not understand that they are determining how many hundreds are in twelve hundred. Number sentences need to be rehearsed and read regularly by teachers. Students must also learn to verbalize number and operational language.

8. 82 ÷ 0.001 (Grade 6/SCO B10)

Only 23 percent of grade 8 students correctly answered 82 000. Most students misplaced the decimal point, indicating that they might have the misunderstanding that division always leads to a smaller answer.

9. 75% × 64 (GRADE 7/SCO B9)

This item was omitted by many students, and only 20 percent correctly answered 48. Students should know that 75% is the same as $\frac{3}{4}$, or 25% ($\frac{1}{4}$) three times. The flexibility to change a value from one form to an equivalent one, which may be easier to work with, is a powerful and necessary tool that can be applied in all numerical tasks.

10. 3%×800 (Grade 7/SCO B9)

Only 28 percent of the students correctly answered 24. Other answers included 2.4, 2400, and 240. Some students included the % sign in their answer. Students need to focus on the meaning of percent and to be thinking about how many hundreds are in eight hundred and triple that amount.

11. $\frac{7}{12} - \frac{1}{4}$ (Grade 8/SCO B6)

Approximately 50 percent of the students correctly answered $\frac{4}{12}$ or $\frac{1}{3}$. However, many students wrote $\frac{6}{8}$, indicating that they were simply subtracting the numerators and the denominators. Using concrete or pictorial representations will help students visualize the operations of fractions.

12. $3\frac{1}{2} \times 5$ (Grade 8/SCO B9)

Only 31 percent of students answered the question correctly. The most common answer given was $15\frac{1}{2}$. Students need to apply the distributive property to obtain $(3 \times 5) + (\frac{1}{2} \times 5) = 15 + 2\frac{1}{2}$, resulting in a final answer of $17\frac{1}{2}$. During instruction, students should have the opportunity to draw pictures and model these kinds of situations, as it will help reinforce the distributive property.

13. 9-0.92 (GRADE 6/SCO B11)

Approximately 40 percent of students answered this question correctly as 8.08. Many students answered 8.8. A good strategy would be to contextualize by having the students think in terms of money.

14. 30%×200 (Grade 7/SCO B9)

Only 40 percent of grade 8 students correctly answered 60. Many students omitted this question; a number of them included the % sign in their answer. Students should be thinking about finding 10% of a number, which can be quickly done, and then triple it.

15. -7-(-5)+2 (GRADE 7/SCO B11)

Almost 45 percent of the students gave the correct answer 0. Incorrect answers included -10 and -4. Using two-coloured counters during instruction would help students visualize integer operations so they become confident and efficient enough to do the procedure mentally.

16. -95÷-5 (GRADE 7/SCO B13)

Approximately 36 percent of grade 8 students correctly answered 19. Common errors included reporting a negative answer and not knowing division facts.

17. 873-98 (GRADE 6/SCO B11)

Only 36 percent of students responded correctly. A significant number of students did not answer this question. A suggested strategy would be to use a compensation technique to subtract 100 from 875 to obtain 775. Mental math requires flexible thinking and adaptive reasoning.

18. 7.3 + 8.7 (Grade 6/SCO B11)

This question was very well answered; 75 percent of students correctly answered 16.

19. $\frac{2}{3} \times \frac{6}{7}$ (Grade 8/SCO B9)

Less than 20 percent of grade 8 students correctly answered $\frac{12}{21}$ or $\frac{4}{7}$. There was evidence that most students incorrectly used a strategy of cross-multiplication, giving answers such as $\frac{18}{14}$ or $\frac{14}{18}$. This clearly illustrates the dangers of relying on rote procedures at the expense of understanding.

20. $\sqrt{144}$ (Grade 8/SCO A2)

Most students answered this question correctly. Approximately 74 percent answered 12. Some students confused square root with taking $\frac{1}{2}$, and answered 72.

My bill was \$64:23. Tax is 15%

The total with tax should be about \$75.00.

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Estimation

What we learned from student responses

Students were required to answer questions by selecting the most appropriate estimate for a computation. They needed to analyse their answers to see if the answers were reasonable. Students needed to consider the features of the problem and choose the appropriate rounding strategy to find an estimate.

- **1.** What is the best estimate for $\sqrt{4+16}$? (Grade 8/SCO A3)
 - (A) 2.4
 - (B) 4.5
 - (C) 6
 - (D) 10

Approximately 55 percent of students correctly selected (B) as the best estimate. Almost 25 percent chose (D), indicating the misconception that the square root means taking one-half. A number of students simply added $\sqrt{4} + \sqrt{16}$ resulting in adding 2 and 4 to get 6, which is answer (C).

- When they go to a restaurant, Mr. and Mrs. Smith leave a tip that is about 15 percent of the bill. Approximately what amount should they leave as a tip when their bill is \$73.58? (GRADE 7/SCO B9, B10)
 - (A) \$7
 - (B) \$10
 - (C) \$13
 - (D) \$15

Less than 33 percent correctly selected (B). An equal number of students incorrectly selected (C), indicating an inappropriate rounding strategy. Finding 10% of the amount, then finding half of that amount (5%) and combining them to get 15% is one possible approach.

3. The product of
$$-\frac{11}{12} \times \frac{7}{15}$$
 is closest to which number? (Grade 8/SCO B9)
(A) $-1\frac{1}{2}$
(B) -1
(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Less than 33 percent of students selected the correct answer (C). Most students incorrectly selected (D), indicating that they ignored the operation sign. Students should have an understanding that $-\frac{11}{12}$ is almost -1 and $\frac{7}{15}$ is almost $\frac{1}{2}$, therefore estimating $-1 \times \frac{1}{2}$, as $-\frac{1}{2}$.

- To carpet a rectangular family room that is 4.8 m by 8 m, the MacKays have chosen a carpet that costs \$30.50 per square metre (tax included). Which is the best estimate of the cost of this carpet? (GRADE 7/SCO B4)
 - (A) less than \$400
 - (B) \$500 to \$600
 - (C) \$1000 to \$2000
 - (D) more than \$2000

Forty-five percent of students correctly selected (C).

- While practising basketball throws, Mary notes that she throws the basketball into the basket 25 times out of 35 tries. Approximately what percent did she throw the basketball into the basket? (GRADE 7/SCO B8)
 - (A) 25%
 - (B) 50%
 - (C) 75%
 - (D) 90%

Approximately 70 percent of students correctly answered (C). Most students recognized that $\frac{25}{35}$ is more than one-half. A number students incorrectly selected (D), indicating that they had difficulty recognizing whether $\frac{25}{35}$ was closer to 75% or 90%. Using benchmarks such as 0, $\frac{1}{2}$, and 1 as 0, 50%, 100% helps students with their reasoning.

regular basis.

- 6. Which is the best estimate for 0.499×0.0038 ? (Grade 7/SCO B1)
 - (A) 2.0
 - (B) 0.2
 - (C) 0.02
 - (D) 0.002

This question was correctly answered by 40 percent of the students. The understanding of place value should lead to students selecting (D) as the most reasonable. They should recognize that the estimate is about half of 0.004, giving 0.002. Another strategy would be seeing that 0.0038 is close to 0, therefore, the choice having a value closest to 0 would be the appropriate estimate.

- 7. What is the best estimate for $\frac{5}{8} + \frac{3}{7} \frac{2}{4} + \frac{8}{9} \frac{1}{8}$? (GRADE 7/SCO B6)
 - (A) 0.5
 - (B) 1.0
 - (C) 1.5
 - (D) 2.0

Approximately 40 percent of students selected the correct answer (C). Students need to identify when fractions are close to 0, $\frac{1}{2}$, and 1 and use these as benchmarks for estimating.

- **8.** Which is the best estimate for $15.987 \div 0.019$? (Grade 7/SCO B1)
 - (A) 800
 - (B) 80
 - (C) 8
 - (D) 0.8

This question was very challenging for the students. Approximately 25 percent selected (A), the correct answer. If students chose values less than 16, there might be a misunderstanding that division always results in a smaller value. Reasoning such as thinking in terms of money (how many groups of two pennies are contained in \$1 and then multiply that number by 16) would help students understand the meaning of division and find an appropriate estimate.

9. Ray's dad takes \$20.00 to the store to pick up some milk and bread. He chooses two packages of milk that cost \$6.39 per package and four loaves of bread that cost \$1.45 each. Did Ray's dad have enough money to buy the items? Approximately how much money did he need to purchase the items? (Show the series of rough calculations you made to approximate the cost of purchasing the items.) (GRADE 7/SCO B1, B2)

Only 16 percent of students received full value for this question. Sixty-five percent of the students received 0 points, because they did exact calculations and then rounded the answer of \$18.58 to \$19.00.

The Jackson family is going to Ontario next summer. They know that they will be driving about 3500 kilometres altogether. Their car uses 10.6 litres of gas for every 100 kilometres. If the average cost of gas is about \$0.75 per litre, approximately how much should the Jackson's budget for gas for this trip? (Show the series of rough calculations you make below.) (GRADE 7/SCO B4)

This question was very poorly answered with 75 percent of students receiving 0 points. Only 11 percent provided a reasonable estimate to receive full value. There is reason to believe that students do not understand the meaning of "estimate your answer."



Examples of student work

Example A



This response was given a mark of 0: incorrect reasoning and wrong answer.

Example B

This response was given a mark of 1.5. The student estimated ($10 \times 0.75 = \$7.50$), but used exact numbers to calculate 7.50 × 35. Although the recording of the equation is not mathematically correct (10×0.75 is not equal to 7.50 × 35), for the purpose of this assessment, the student was still given 1.5. However, it is expected that these types of mathematical inaccuracies would be addressed in a classroom situation.

Example C

3500 Km 10.00 = 100 Km 102 for every 100 Km \$1.00 litre 100.00 = 1000 Km 350.00 = 3500 Km 350.00

This response was given a mark of 2: appropriate rounding, work well organized, and the correct answer. Again, the recording of information such as 10.00 = 100 km, when showing their work, is mathematically incorrect and must be discussed and discouraged in a classroom situation. Students should also be encouraged to write their answer in sentence form.

Paper-and-Pencil Procedure Questions

What we learned from student responses



Students were given eight computational tasks requiring paper-and-pencil procedures to solve them. Each task was assigned a value of 2 points.

1. $3\frac{1}{2} \div 1\frac{1}{4}$ (Grade 8/SCO B7)

Less than 18 percent correctly answered the question. Eighty percent of students received a mark of 0. Many students found the lowest common denominator and then added, instead of dividing, incorrectly applying rote rules and procedures. Students need more practice with conceptual and procedural understanding involving division of mixed numbers.

2. 16.45×0.4 (Grade 6/SCO B2)

Approximately 33 percent of the students correctly answered this question. A large number of students incorrectly placed the decimal point in the answer.

3. 5(-4.2 + 2.16) (Grade 8/SCO B12)

Only 13 percent of grade 8 students correctly answered this question. Students had difficulty evaluating this expression, writing values such as 2.14 or 6.36 inside the parentheses, suggesting difficulty with rational number operations. Another identified error was that students added instead of multiplying by 5, not realizing the implied multiplication.

4. [3.9 - (-2.12)] ÷ (-2) (Grade 8/SCO B12)

About 16 percent correctly did the computations. Students often added tenths to hundredths, giving 5.21 inside the parentheses. Another common mistake was obtaining the answer as a positive number. This incorrect sign error was noticed in almost all tasks involving negative and positive values.

5. 120% of 43 (GRADE 8/SCO A8)

Almost 33 percent of grade 8 students did the computation correctly. Students gave answers that were less than 43, therefore, not attending to the reasonableness of their answer. Students need to ask themselves if their answer makes sense. If 100% of 43 is 43, could 120% be less than 43?

6. Solve for n: $\frac{5}{n} = \frac{6}{10}$ (Grade 8/SCO A9, B2)

Approximately 12 percent of students answered this question correctly. The great majority of the students made no attempt to solve the proportion. Some students used cross-multiplication to solve this problem. Although it is a correct method for solving this equation, it is to be discouraged, as students often do not have an adequate understanding of why it works. A better approach for solving this problem may be to use the following reasoning:

Step 1: Determine the multiplier.

What number multiplied by 6 will give you 5? $6 \times \frac{5}{6} = 5$ Step 2: Multiply the denominator by the same multiplier. $10 \times \frac{5}{6} = \frac{50}{6}$ or $8\frac{1}{3}$

> Therefore $n = 8\frac{1}{3}$ OR

Multiply each side of the equation by 10n, which will eliminate the denominators. $\frac{5}{n}(10n) = \frac{6}{10}(10n)$

$$50 = 6n$$
$$\frac{50}{6} = \frac{6n}{6}$$
$$\frac{50}{6} = n$$
$$n = 8\frac{1}{3}$$

7. Solve for x: 5(2-3x) = -20 (Grade 8/SCO C6)

Only 10 percent of students correctly answered this question. Many students chose not to divide each side of the equation by 5, and when they attempted to remove the parentheses, incorrectly wrote 10-3x. Students need to be introduced to solving equations using concrete materials in order to have a good understanding of the distributive property and to be able to connect this understanding to the symbolic solution.

Did you know

Slightly more than half of teachers reported that their students engage in writing tasks related to mathematics. 8. $3\frac{1}{4}-\frac{2}{3}$ (Grade 8/SCO B5)

Thirty-three percent of grade 8 students answered this question correctly. Many students wrote equations such as: $\frac{13}{4} - \frac{2}{3} = \frac{39}{8}$. Again problems such as this arise when students are taught rules like "cross-multiply" without adequately understanding when to use it and why it works.

Another common answer was: $\frac{13}{4} - \frac{2}{3} = 11$. Students seem to confuse the rules for operations with fractions, which indicates that students do not have the necessary conceptual and/or procedural understanding to work with fractions.

Number Concepts and Operations (GCO A AND GCO B)

Students responded to both selected-response and constructed-response questions. The questions assessed students' knowledge and comprehension of integers, positive and negative decimals, common fractions, ratios, proportion, and percent. These questions also assessed understanding of procedural knowledge and application.

The following graphs show the provincial and individual school board mean percentage score for Number Concepts (GCO A) and Operations (GCO B).



Selected-Response Questions

What we learned from student responses

- In a survey of 1000 Nova Scotians, 600 people indicated they preferred Sunday shopping, while the rest did not want Sunday shopping. Which of these represents the ratio of those who prefer Sunday shopping to those who do not want Sunday shopping? (GRADE 6/SCO A3)
 - A) 3:5
 - B) 5:3
 - C) 3:2
 - D) 2:3

Approximately 25 percent selected the correct answer (C). Some students seemed to incorrectly interpret the ratio, as they chose (D) for their answer. Many students simply used the numbers, 1000 and 600, given in the problem and gave 3:5 as their answer.

2. Which of the following is NOT true about this grid? (GRADE 7/SCO A10)



- A) 0.65 is shaded
- B) 0.65% is shaded
- C) $\frac{13}{20}$ is shaded

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D) 65% is shaded

Approximately 44 percent of the students selected the correct answer (B). Response (C) was selected by more than 33 percent of the students, indicating that they did not recognize that 65% and $\frac{13}{20}$ represent the same amount.

Ron drew the following number line to represent a product. Which product is he representing? (GRADE 8/SCO B8)



Almost 40 percent of students correctly selected (C) as the answer. A significant number of students selected (D) as the answer, indicating that perhaps they were counting the actual points instead of the distance on the number line.

- **4.** Jim must multiply an amount by 0.15 to determine how much sales tax to pay. Of the fractions $\frac{15}{100}$, $\frac{3}{20}$, or $\frac{18}{120}$, by which could he multiply instead? (GRADE 6/SCO A9)
 - A) by $\frac{15}{100}$ only B) by $\frac{3}{20}$ only C) by $\frac{18}{120}$ only
 - D) He could multiply by any one of them.

This question was correctly answered by 60 percent of students, who selected the correct answer (D). Almost 30 percent selected (A), and, therefore, they may not have looked at the other choices. When solving multiple choice questions, it is important that students consider all choices before making a selection.

- 5. A chocolate bar is divided into 8 equal pieces. Barbara ate 3 of the pieces and John ate 2 of the pieces. What fraction represents the amount remaining? (GRADE 8/SCO B11)
 - A) $\frac{3}{8}$ B) $\frac{5}{8}$ C) $\frac{1}{4}$ D) 3

Almost 75 percent of the students correctly selected (A) as the answer. The remaining students generally selected (B), possibly indicating that they misread or misunderstood the question.

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- 6. In a class of 28 students, there are 12 boys. Which of the following represents the ratio of girls to boys in this class? (GRADE 6/SCO A3)
 - A) 4:7
 - B) 3:7
 - C) 4:3
 - D) 3:4

Less than 50 percent of grade 8 students answered this question correctly by selecting (C). The most common error was reversing the ratio and reporting (D) as the answer.

7. Which of the points indicated on the number line below most likely indicates the point corresponding to $-0.5 \div -0.25$? (GRADE 8/SCO B12)

Q ↓			R ●────	S		T I		*
-2		-1	0		1	2	3	-
A)	Q							
B)	R							
C)	S							
D)	Т							

Less than 50 percent of the students were able to answer this question correctly by selecting (D). Many students chose a negative value, supporting earlier indicators that they have difficulty with rational number operations and determining the correct sign for their answers.

 Which product of two common fractions is represented by this picture? (GRADE 8/SCO B8)



Almost 50 percent of the students were able to connect the picture to the symbolic operation and, therefore, correctly answered (C).

- Which of the following computations will have a negative integer as the answer? (GRADE 7/SCO B12)
 - A) $5 \times (-3) \times (-4)$
 - B) $(-3) \times (-5)$
 - C) $(-2) \times (-5) \times (-6)$
 - $D) \quad (-5) \times (-3) \times (-2) \times (-1)$

More than 80 percent of teachers reported that their students used calculators at least once a week.

This question was well answered, as almost 70 percent selected the correct answer (C). It is interesting to note that students were able to determine the correct sign in this type of task, yet did not seem to make the connection when required to apply that knowledge in other problem-solving tasks.

- You are constructing a circle graph to show how many days you go to school each year. You know you go to school 180 days. Which of the following proportions can you use to calculate the number of degrees in the sector that represents this data? (GRADE 8/SCO A9)
 - A) $\frac{1}{180} = \frac{n}{360}$ B) $\frac{180}{365} = \frac{n}{360}$ C) $\frac{180}{365} = \frac{360}{n}$
 - D) $\frac{180}{360} = \frac{n}{360}$

Almost 45 percent of students correctly selected (B) as the answer. The remaining three choices were selected by approximately the same percentage of students. These results support earlier indications that students struggle with proportional reasoning.

11. Which of the following fractions is equivalent to (0.2 x 0.8)?

(GRADE 7/SCO A10, GRADE 8/SCO B1)

A)	$\frac{4}{25}$
B)	$\frac{16}{10}$
C)	$1\frac{6}{10}$
D)	$\frac{4}{20}$

Less than 30 percent of students correctly selected answer (A). Most students believed that 0.2×0.8 was $\frac{16}{10}$. Students need many opportunities to make connections between decimals and fractions.

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Constructed-Response Questions

What we learned from student responses

John is going to explain to Ben why (-3)-(-5) = +2. If ■ represents +1, and □ represents -1, draw a picture to show how John used these tiles to explain the answer to Ben. (GRADE 7/SCO B11)

This question was marked out of 3 points. Only 10 percent of students received full value for the question. Some did drawings, showing no understanding of subtraction and using the zero principle. Students should be introduced to two-colour counters to understand integer operations.

Here is a possible pictorial representation that John could have used.




2. You are given two common fractions: $\frac{8}{9}$ and $\frac{11}{12}$. Which fraction is larger? Explain in words how you could decide without changing the fractions to decimal numbers or finding a common denominator.

Almost 50 percent of the grade 8 students correctly chose $\frac{11}{12}$, but were unable to provide an accurate explanation. This question was valued at 2 points; 11 percent of the students received full value. More than 40 percent of students received 0 for this question. Students were expected to recognize that $\frac{11}{12}$ is larger because $\frac{1}{12}$ is smaller than $\frac{1}{9}$; therefore, $\frac{11}{12}$ is closer to 1 than $\frac{8}{9}$.

3. David is not sure how to subtract $\frac{2}{3}$ from $\frac{3}{4}$. Draw a picture to show how David would obtain the answer. Record your answer to the subtraction on the line below.

Only 10 percent of students received full value for this question, and more than half received 0. Many students wrote $\frac{2}{3} - \frac{3}{4}$ instead of $\frac{3}{4} - \frac{2}{3}$. A common error was comparing $\frac{2}{3}$ and $\frac{3}{4}$ from wholes that were clearly not drawn identical in size. Students need to connect concrete, pictorial, and symbolic representations when doing operations with fractions.

Here is a possible pictorial representation of this operation.



- **4.** The Bridgewater Pizza Shop makes a deluxe pizza that sells for \$10.80. The owner decides to sell the pizza by the slice. She can cut the deluxe pizza into 12 slices and she can also cut the pizza into 8 slices.
 - a) To make \$10.80 on each pizza sold, how much should she charge for a large slice? (Show your work.)
 - b) If she wants to charge \$0.45 for a slice and still make \$10.80 for the whole pizza, she cannot cut it into 8 or 12 pieces.

Into how many slices should she cut the pizza? (Show your work.) (GRADE 8/SCO B13)

More than 50 percent of the students correctly answered this question. A common error was incorrectly writing the number sentence as $12 \div \$10.80 = \0.90 .

5. Janet was given a word problem to solve. She made the calculations shown below in order to find the final answer of \$6.68.

Examine Janet's calculations and create a reasonable word problem that she might have been solving. (GRADE 8/SCO B13)

 $2 \times \$3.99 + 5 \times \$1.89 = \$17.43$ $15\% \times \$17.43 = \2.61 $\frac{(\$17.43 + \$2.61)}{3} = \$6.68$

This question was marked out of three points. Approximately 26 percent of students received full value. Some of the errors that students made were simply rewriting the problem in words and not giving their answer in the form of a question. Students need practice in writing story problems requiring three-step computations.

Rubric

The following rubric was used to mark this problem.

- **3 points:** A story problem that includes all the given information, asks a question, and provides a reasonable context; givens:
 - A) 2 of something, each of which is \$3.99 added to 5 of something at \$1.89 each
 - B) Add tax of 15%
 - C) Divide total by 3
 - D) Question asked
- **2 points:** A reasonable context, but either one of the givens was left out OR was incorrectly connected to the story
- 1 point: Attempted a problem that has some of the givens
- **0 points:** Other attempts
- 9: No attempt

Examples of student work

Example A

Finet bought 2 cospetts and they cost 3. 99 even and five Itichers which are 1. 89 each so those totals= 17.43 now she finds out the tox on it 00 15% mutliplies by 17.43 is 2.6 = \$6.68 Qa 17.43 + 2.61

This response was given a mark of 0. The student simply put the solution in a context and did not write a word problem to be solved.

Example B

Janet and her-three friends decide to buy a snack. They buy two bags of chips which are 3.99 each and the 5 bottles of pop which are 1.89 each This all eagules 17.43 plus tax which is 2.61. This all comes to 20.04, then because all three girls want to eat the snack they devide the cost by three. That comes to 16.68

This response was given a mark of 0. Again the student simply added a context for a solution instead of writing a word problem.

Example C

If Three people chipped in to buyd bottles of pop for\$3.99+ tax each and 5 candy bars at \$1.89 + tax each.

This response was given a mark of 2. The student included all of the givens, but did not ask a question.

Example D



This response was given a mark of 3. All of the givens were included, and a question was asked.

Patterns and Relationships (GCO C)

The tasks in this strand assessed students' abilities to recognize and represent patterns and relationships, and to create and solve algebraic equations. Students responded to selected-response and constructed-response items.

Students were required to answer questions that included representing, analysing, and generalizing patterns with tables, graphs, words, and symbols. The following graph shows the provincial and individual school board mean percentage score for Patterns and Relationships (GCO C).



Selected-Response Questions

What we learned from student responses

 If the shaded shapes represent positive quantities, what algebraic expression is represented by this algebra tile display? (GRADE 8/SCO B15)



- (A) $2x^{2} + 4x + 4$ (B) $2x^{2} - 6x - 4$
- (C) $2x^2 2x 4$
- (D) $2x^2 + 3x 10$

Approximately 65 percent of students correctly answered (C), showing that most students were able to interpret a pictorial representation of algebra tiles.

Look at the displays made by Charles. The number of dots in each display forms a pattern. If Charles were to continue the pattern, which of the following numbers will NOT fit into Charles' pattern? (GRADE 6/SCO C7)

- (B) 31
- (C) 47
- (D) 57

Approximately 50 percent of students correctly identified that 57, (D), did not fit into the pattern. A significant number of students simply selected the next term in the sequence by answering (A).



- **3.** The cost of a taxi trip C, in dollars, for n minutes, is given by the formula C = 0.12n + 4.50. Which statement would always be true for the relationship in the formula? (GRADE 8/SCO C1)
 - (A) The cost for a taxi trip is 12 cents per minute.
 - (B) The cost for a taxi trip is always \$4.50.
 - (C) Each minute results in an extra cost of 12 cents.
 - (D) Each minute is twelve more than the previous minute.

Less than 50 percent of the students were able to correctly interpret the equation and match it with the correct statement (C). Students should be given plenty of opportunities to interpret linear relationships and the meaning of the variables and constants within a context. These opportunities will provide students with the necessary foundational ideas to make the connections to other algebraic concepts.

The table below shows the monthly phone bill for the Smith family home, in 2003. Which graph shows the cost for the phone use for the Smith family in 2003? (GRADE 8/SCO C2)

2003 Monthly Payments

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Phone Bill (in \$)	22.61	24.25	31.02	27.34	29.92	33.10	36.14	35.50	34.01	31.05	36.20	40.12



Almost 90 percent of students correctly answered (B). The students did very well interpreting the table and relating to the correct graph.

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5. Given the table, which graph below correctly represents the line containing the ordered pairs of numbers in the table? (GRADE 8/SCO C3, C4)



Only about 33 percent answered the correct response (A). Students did poorly determining which linear graph corresponded to the table of co-ordinate points. It may have been the lack of context that caused the poor response.

Constructed Response Questions

What we learned from student responses

 Bob used white and grey square tiles to create this pattern. (GRADE 7/SCO C2, GRADE 8/SCO C1)













(A) Complete the table for Bob's pattern.

Display number	Number of white tiles	Number of grey tiles	Total number of tiles
1			
2			
3			
4			

(B) Look for patterns in the table in part (A) so you can continue the table without having to draw Bob's pattern.

Display number	Number of white tiles	Number of grey tiles	Total number of tiles
5			
10			
100			
n			

Did you know ...

More than 60 percent of teachers feel that parents do not understand why manipulatives are used to develop concepts. (C) Bob noticed the perimeter of Display #1 is 10 units and the perimeter of the Display#2 is 18 units. Complete the following table for Bob and look for a relationship so that you can find the perimeter without counting.

Display number	1	2	3	4	5	6	10	100
Perimeter	10	18						

Write a rule for finding the perimeter of any display without counting.

Write your rule in the form of an equation.

Correct answers

(A)	Display number	Number of white tiles	Number of grey tiles	Total number of tiles
	1	1	3	4
	2	2	6	8
	3	3	9	12
	4	4	12	16

(-)				
(B)	Display	Number of	Number of	Total number
	number	white tiles	grey tiles	of tiles
	5	5	15	20
	10	10	30	40
	100	100	300	400
	n	n	3n	4n

(C)	Display number	1	2	3	4	5	6	10	100
	Perimeter	10	18	26	34	42	50	82	802

Rule for finding the perimeter of any display without counting: P = 8n + 2

This question was marked out of 6 points. Approximately 2 percent of the students received full value, while 60 percent of the students received 2 points. Students generally answered parts (A) and (B) correctly, except when the variable n was introduced. Many students had difficulty seeing the pattern for perimeter in the table in part (C). Very few were able to determine a rule for finding perimeter and wrote 2L+2W. The students seemed unable to determine the role of a variable and unable to distinguish between an equation and an expression. Again, it is very important for students to be given opportunities to generalize a pattern, giving them the required skills to better understand more advanced algebraic concepts and procedures.

- 2. For the pictures you will draw in this question, use shaded tiles to represent positive values and unshaded tiles to represent negative values. (GRADE 8/SCO B16)
 - (A) Margo says that 3(2x-1) is 6x-1. Draw a picture of the algebra tile display that you would make to show 3(2x-1).
 - (B) Is Margo correct in her claim? Explain using your picture drawn in part (A).
 - (C) Margo wants to add and subtract these two expressions $(x^2 + x)$ and $(-2x^2 + 3x)$
 - (i) Show the algebra tile display Margo would need to add the two expressions. $(x^2 + x) - (-2x^2 + 3x)$

The sum of the two expressions is: _____

(ii) Show the algebra tile display Margo would need to subtract the two expressions. $(x^2 + x) - (-2x^2 + 3x)$

The difference of the two expressions is: _____

Most students attempted the problem, but only 1 percent of the students received full value for the question. Many showed little understanding of how to draw a picture for 3(2x-1) and often based their explanation on memorized rules instead of interpreting 3(2x-1) as three sets of 2x-1. Students were generally able to represent the addition of two polynomials, but most were unable to represent the subtraction of polynomials. Most students relied on rules such as "flipping the tiles" or "changing the signs," which does not necessarily show an understanding of polynomial operations.

Examples

Here is an example of a possible answer for each question.

(A) 3(2x-1)



(B) We can see that we have 6x-3, so Margo is not correct.

id you know ...

Almost 40 percent of teachers disagreed or strongly disagreed with the statement "Most students enjoy doing mathematics." (C) (i) $(x^2 + x) + (-2x^2 + 3x)$



The sum of the expression is: $-1x^2 + 4x$

(ii)
$$(x^2 + x) - (-2x^2 + 3x)$$



To remove $-2x^2 + 3x$, use the zero property.



The difference of the two expressions is $3x^2 - 2x$

Measurement and Geometry (GCO D AND GCO E)

Students were asked questions that required analysis of two- and three-dimensional geometric shapes. They were required to apply transformations and use symmetry to analyse mathematical situations. Students must be able to use visualization, spatial reasoning, and geometric modelling to solve problems.

Students need to understand measurable attributes of objects and the units and processes of measurement. This assessment required students to answer questions that included angle measurement, perimeter, area, surface area, and volume.

The graph below shows the provincial and individual school board mean percentage for measurement and geometry concepts. (GCO D, GCO E)



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Continuing a Coherent Mathematics Program A Study Document for Educators at the Junior High Level

Selected-Response Questions

What we learned from student responses

- A package of paper weighs 2 kilograms and contains 500 sheets of paper. How many grams does one sheet of paper weigh? (GRADE 7/SCO D1)
 - (A) 0.02g
 - (B) 0.04g
 - (C) 2g
 - (D) 4g

Approximately 25 percent of the students selected the correct answer (D). Most students selected (A). Many students showed difficulty in doing the conversion from kilograms to grams.

2. Examine the measurements in the right triangle below. What is the length of side XY? (GRADE 8/SCO D10)



This question was well done, with almost 60 percent of students correctly answering (C). Many students selected (B), which is the measure of segment XY using a ruler. Students should be familiar with the Pythagorean triple, 3-4-5, and recognize that 12-16-20 is a multiple of this triple.

3. If the shape on the co-ordinate plane below undergoes a dilatation of factor 2 with centre at Q, what will be the area of the dilatation image? (GRADE 8/SCO E5)



12 square units (A)

- (B) 18 square units
- (C) 36 square units
- 72 square units (D)

More than onethird of teachers feel that the grade 8 mathematics curriculum is too difficult for most of their students.

Only 25 percent of students correctly selected (C) as the answer. Most students simply doubled the area. Students need to explore how perimeter and area change when a shape undergoes a dilatation.

Triangle DEF is an isosceles triangle. Determine the measure of $\angle E$. 4. (GRADE 6/SCO E3, GRADE 7/SCO E7)



More than 50 percent of the students correctly selected (C). This question requires basic knowledge of the properties of an isosceles triangle and that the sum of the angles of a triangle is equal to 180°. It is important to provide students with opportunities to make connections between various geometric properties to solve a problem. This type of spatial reasoning lays the foundational ideas for more formal geometric proofs.

Two rectangles ABCD and LMNO are shown below on the grid. The area of rectangle 5. LMNO is what percent of the area of rectangle ABCD? (GRADE 8/SCO D1)

50%

 $66\frac{2}{3}\%$

150%



Almost 50 percent of the students correctly selected (C). A significant number of students selected (B), implying that they compared the smaller area to the larger. Students need to see more situations where the percentage is greater than 100%.

Which of the following answers is the correct volume of the cylinder below? The answer has been rounded to the nearest unit. (GRADE 8/SCO D7)



Less than 20 percent of students correctly answered (C). More than 50 percent of students multiplied the diameter by the height to obtain an answer. Students need experience in manipulating three-dimensional shapes to gain a better understanding of the concept of volume as well as how to measure it.

- A large bug measures 435 millimetres. What is the length of this bug in metres? (GRADE 7/SCO D1)
 - (A) 0.435 metres
 - (B) 4.35 metres
 - (C) 43.5 metres
 - (D) 435 000 metres

More than 60 percent of the students correctly selected (A).

8. If the measure of $\angle ROS = 30^\circ$, what is the measure of $\angle SOT$? (Grade 7/SCO E4)



Almost 70 percent of the students correctly selected (B) as the answer.

9. What is the measure of ∠A in this triangle? (GRADE 6/SCO E3, GRADE 7/SCO E7)



More than 60 percent correctly answered the question by selecting (C). A significant number of students answered (B), implying that they subtracted 47° from 90°.

10. In the diagram below, there are two parallel lines cut by a transversal. What is the measure of the angle indicated by "x"? (GRADE 7/SCO E5, E6)



Most students can calculate the angles of parallel lines cut by a transversal. Almost 60 percent correctly selected (D). More than 25 percent of the students selected (C); yet an answer of 80° should appear unreasonable, as the diagram shows an angle greater than 90°.

The figure below shows a square inscribed in a larger square. What is the area of the smaller square inscribed in the larger square? (GRADE 8/SCO D10)



- (A) 36 square units
- (B) 64 square units
- (C) 100 square units
- (D) 196 square units

id you know ...

Eighty-nine percent of teachers agree or strongly agree with the statement "My major challenge in teaching grade 8 is teaching all the outcomes."

Approximately 25 percent of the students answered this question correctly, by selecting (C). Students might have solved this question using the Pythagorean Theorem. Students should be able to recognize that a 3-4-5 triangle is a right triangle and should, therefore, see 6-8-10 as a right triangle.

Constructed-Response Questions

What we learned from student responses

 The Yang family is setting up a train set. They use a rectangular plywood base for the train that will be 95 cm by 155 cm. They want to put wood trim around the plywood base. The trim costs \$6.59 a metre plus 15 percent tax. How much wood trim will the Yang family need to buy and how much will it cost? (Show your work.) (GRADE 6/SCO B8, GRADE 7/SCO B10, GRADE 8/SCO D1, D4, D5)

This question was marked out of 3 points. Only 12 percent of students received full value, and approximately 50 percent received a mark of 0. Some of the common errors that students showed in their work included

- not including units in their answer
- incorrectly finding perimeter using only two sides
- not distinguishing between perimeter and area (i.e., some students multiplied the length by the width to find perimeter)

Many students lacked organization in presenting their solution. Students must be reminded that when communicating a solution, it must be done in a clear, precise, and well-organized manner.

Rubric

The following rubric was used to mark this problem:

3 points: Finds the correct solution showing all the work

(Perimeter: 5 x \$6.59 = \$32.95 and \$32.95 x \$1.15 = \$37.89 or \$37.90)

- 2.5 points: Everything is correct except for conversion of units
- **2 points:** One error in determining the cost of the trim or calculation of tax or error in calculating perimeter (Ex. P = 2.5 m or 250 cm)
- 1 point: Two errors or used area rather than perimeter
- **0 points:** Other attempt (more than 2 errors)
- 9: No attempt

Examples of student work

Example A

\$ 6.59 × 95 = 626.05 6.59 × 95 = 626.05 6.59 × 155 = 1021.45 6.59 × 155 = 1021.45 4547.1

This response was given a mark of 0. The student had more than two errors: no units identified, did not make the conversions, and did not calculate the tax.

Example B



This response was given a mark of 1. This student used area instead of perimeter.



Example C

$$\frac{450 \text{ cm}}{100} = 2.5 \text{ m} \quad 2.5 \text{ m} \quad 3.5 \text{ m} \quad 100 = 3$$

This response was given a mark of 2. The student found the perimeter using only two sides.

Example D

This response was given a mark of 3. The student was able to correctly solve the problem. Note: Although the student was given full value for understanding and correctly solving the problem, the presented solution lacked organization. Inattention to details should be discouraged and addressed in a classroom situation.

Example E



This response was given a mark of 3. The student was able to correctly solve the problem and recorded the solution in a clear and organized manner. Students should also be encouraged to write their answer in sentence form. Examine the plan for Ms. Herns' new flower garden, shown below. Using the measurements given on the plan, find the area of the garden. (Show your work.) (GRADE 8/SCO D6)



This question was marked out of 3 points. Less than 10 percent received full value, while more than 50 percent received 0 out of 3. Despite the fact that students had access to calculators, many students made computational errors. Most students failed to use units in their answer. A number of students simply added the numbers given. In about 15 percent of the answers, students were able to determine the area of the rectangular region, but very few students managed to find the area of the quarter circle or the triangle.

Rubric

The following rubric was used to mark this problem:

- **3 points:** Correct answer of 29.14 m² with the three sub-regions (1/4 circle, rectangle, and triangle)
- 2.5 points: Omitted square metres in the answer or one computational error
- 2 points: Found two areas of the two sub-regions correctly
- 1.5 points: Same as "2," but with a computational error OR missing unit
- 1 point: Found the area of one sub-region
- 0.5 point: Same as "1," but with a computational error or incorrect unit
- 0 points: Other attempts (or no work shown)
- 9: No attempt

Examples of student work

Example A



This response was given a mark of 0. The student simply added the numbers.

Example B



This response was given a mark of 1. The student was able to find the area of only one subregion.





This response was given a mark of 2. The student correctly found the area of two subregions.

Example D



This response was given a mark of 2.5. The student has the correct process, but made one computational error. (5 + 2 = 8)

Example E



This response was given a mark of 3. The student correctly solved the problem.

3. On the co-ordinate system below, draw the image of the given triangle ABC under a reflection in the y-axis. Label the image A'B'C'. Rotate the image 90° counterclockwise about the origin. Label this image A"B"C". After completing each transformation, indicate the co-ordinates of the vertices in the space provided below.
(GRADE 6/SCO E10, GRADE 7/SCO E9)



This question was marked out of 2 points. Less than 5 percent of the students received full value, and almost 50 percent received 0 out of 2. Approximately one-third of the students could reflect a triangle over the y-axis. Although students were permitted to use tracing paper, most could not do a 90° rotation. Some did a 90° rotation using point A instead of the origin as the centre of rotation.

Rubric

The following rubric was used to mark this problem:

2 points: Correct final result shown: triangle with vertices A"(1, -2), B"(-2, -3) and C"(-1, -5) with or without intermediary triangle

1.5 points: Correct reflection and rotation with one error

1 point: A correct reflection in the y-axis with A' (-2, -1), B' (-3, 2) and

C' (-5, 1), accept correct reflection on graph even if co-ordinates are not indicated or are incorrect

0 points: Wrong answer

9: No attempt

Examples of student work

Example A



This response was given a mark of 0. The reflection and the rotation are both incorrect.

Example B



This response was given a mark of 1. The student did a correct reflection.

Example C



This response was given a mark of 1. The student did a correct reflection. The rotation is a 90° rotation, but not about the origin.

Example D



This response was given a mark of 2. Both transformations are correct.

- **4.** (A) Draw a line segment that is 8 cm long.
 - (B) Draw a line segment that is 175% of the line segment drawn in part (A).
 - (C) Draw a line segment that is 75% of the line segment in part (A).
 - (D) The largest line segment is what percent of the shortest line segment?

This question was marked out of 4 points, and almost 10 percent received full value. Some students started measuring at 1 cm on the ruler and therefore drew a 7cm line segment. Other students started at the end of the ruler before "0." Very few students were able to determine what percentage of the shortest line segment is the longest line segment.

5. Josh made an interesting 3-D shape using 9 cubes. The orthographic views of his shape are:



Draw an isometric picture of Josh's shape on the isometric paper below. An example of a cube is shown to remind you of isometric drawings. This cube is not part of your drawing. (GRADE 6/SCO E2, GRADE 7/SCO E8, GRADE 8/SCO E1)



This question showed great improvement, when compared to results on the previous provincial assessment. An equal number of students, approximately 40 percent, received either full value or 0 points for the answer.

Data Management and Probability (GCO F AND GCO G)

In this assessment, students were expected to interpret and create various graphical representations of data, including histograms, stem-and-leaf plots, box plots, and scatter plots. Students were expected to be able to find, use, and interpret measures of central tendencies including mean, median, mode, and range.

Students are expected to be able to determine probabilities for simple compound events, using such methods as organized lists and tree diagrams.

The graph below shows the provincial and individual school board mean percentage score for Data Management and Probability concepts (GCO F, GCO G)



Selected-Response Questions

What we learned from student responses

 The stem-and-leaf plot below shows the ages of the people who bought bicycles at a store during a sale. What is the median age of the people who bought bicycles during the sale? (GRADE 6/SCO F4, F7, F8)

Ages of People

Stem	Leaf	(A)	16
1	134556668	(B)	24
2	001245566	(C)	245
3	4459	(C)	24.5
4	336	(D)	29
6	13		
7	1		

Thirty-three percent of students correctly selected (C) as the answer. Almost an equal number confused median with mode and selected (A). A significant number of students determined the mean of the data.

Four grade 8 classes collected money for a fund-raiser. The results are provided in the graph below. Based on the information given in the graph, which of the following statements is true? (GRADE 6/SCO F4, SCO F7)



- (A) Class 8A collected the most money.
- (B) Class 8D collected more money than class 8A.
- (C) Class 8C is the class with the most students.
- (D) Class 8B collected approximately half as much money as did class 8D.

Almost 80 percent selected (D), the correct answer. Most of the remaining students selected (C), indicating a misread of the horizontal axis.

- 3. Brad and Lauren are playing a game in which they are each tossing two regular dice with faces numbered from 1 to 6. If the numbers match on Brad's two dice, he gets a point. If the number on Lauren's dice do not match, she gets a point. Which player has the greatest chance of winning the game? (GRADE 7/SCO G4, GRADE 8/SCO G2)
 - (A) Brad
 - (B) Lauren
 - (C) They both have the same chance of winning.
 - (D) There is no way to tell.

Most students correctly selected (B). Out of 36 possible outcomes, there are only 6 chances of having doubles.

) Did you know .

Approximately 90 percent of teachers feel they are well prepared and confident in teaching grade 8 mathematics.

Constructed-Response Questions

What we learned from student responses

- Suppose Joe has two coins. He tosses the first and then tosses the second. (GRADE 7/SCO G3)
 - (A) Draw a tree diagram to show all possible outcomes of Joe's tossing of the two coins.
 - (B) What is the probability that Joe will get two heads as a result of tossing the two coins?

This question was valued at 2 points; less than 10 percent of the students received full value. Very few could draw an accurate tree diagram, but a number of students could identify some of the outcomes. Many students who wrote all four outcomes (HH, HT, TH, TT) answered part (B) as 50 percent, perhaps thinking of flipping one coin.

Answer



Results from tossing two coins: HH, HT, TH, TT

(B) The probability that Joe will get two heads is $\frac{1}{4}$.

Another example for a probability task

The following is another example of a task where students are asked to determine probability outcomes.

Draw a tree diagram showing all possible outcomes when a coin and a die are tossed. What is the probability of getting a head and a prime number?



The outcomes are: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6.

The probability of getting a head and a prime number is: $\frac{3}{12} = \frac{1}{4}$ (H2, H3, H5)

The following box-and-whisker plots show the results of a math test in three grade 8 classes. Decide whether the following statements are true or false. If you decide the statement is false, explain why it is false. (GRADE 8/SCO F5, F6)



a)	The top mark in Ms. Gill's class was 75.	Т	F
b)	One-half the students in Mrs. Smith's class had a mark less than 70.	Т	F
c)	The number of students in Mr. Jones' class who was marked under 65 is the		
	same as the number who was marked over 90.	Т	F
d)	The students in Ms. Gill's class all made lower marks than the students in		
	Mr. Jones' class.	Т	F
e)	The mean mark in Mrs. Smith's class is 70.	Т	F
f)	Twenty-five percent of the students in Ms. Gill's class marked above 75.	Т	F

The question, valued at 4.5 points, required the students to interpret box plots. Only 5 percent of students received 3.5 or more points out of the total. When students were answering false, most students were unable to provide an explanation. A very common misunderstanding was not recognizing that 25 percent of the data is represented by each whisker regardless of its length.
3. In a class of grade 8 students, each person measured in centimetres the distance around his or her head. Every student also measured his or her height. The data collected is presented in table below:

Distance around head (cm)	32	35	32	46	37	43	50	41	38
Height (cm)	96	112	95	128	118	135	158	135	122

(A) Draw a scatter plot of the data and draw a line of best fit. (GRADE 8/SCO F4, F6)



- Distance around head (cm)
- (B) A student had a height of 145 cm. What would you predict to be the distance around the student's head?
- (C) A student measured the distance around the principal's head to be 60 cm. What would you predict to be the principal's height?

This question was valued at 6 points. Less than 3 percent received 6 points, and almost 20 percent received 0 points. (GRADE 8/SCO F4, F6)

Common errors included

- inappropriate scale
- not identifying the relationship between two variables
- unable to predict from the graph, but could predict from the table
- unable to draw a line of best fit

Students need to have more opportunities to construct scatter plots and draw lines of best fit. They need to interpolate and extrapolate data from the graph.

Rubric The follow	wing rubric was used to mark this problem
Part (A)	2 points: Appropriate scale on the horizontal and vertical axis, including an indication of a clear break in the axis, if necessary (remove 0.5 pt if missing); one point for each axis
	1 point: Plotted most of the co-ordinate pairs accurately
	1 point: Drew a line of best fit
Part (B)	1 point: Predicted the distance around the student's head to be approximately 46–50 cm, or correct interpretation based on a line of best fit
Part (C)	1 point: Predicted the height to be about 3 times the distance of 60 cm (175–185 cm)
	9: No attempt

Examples of student work

Example A



The response was given a mark of 2.5 in total.

- Part (A) 0.5 points: Appropriate scale, but no indication of a clear break at the beginning of the axes. The student could not plot coordinate pairs correctly, and there is no line of best fit.
- Part (B) 1 point: Correct answer
- Part (C) 1 point: Correct

Example B



The response was given a mark of 3.5 in total.

- Part (A) 2.5 points: Scaling is correct (although incorrect placement at the origin) on both axes, no clear break in axes, correct plotting of the co-ordinate points, no line of best fit.
- Part (B) 0 points: Incorrect answer
- Part (C) 1 point: Correct answer

Example C



The response was given a mark of 4 in total.

Part (A) 3 points: Correct scaling with clear break on the axes, correct plotting of the co-ordinate points, but no line of best fit.

Part (B) 0 points: Incorrect answer

Part (C) 1 point: Correct answer

Example D



The response was given a mark of 6 in total.

- Part (A) 4 points: Correct scaling with clear break on the axes, correct plotting of the co-ordinate points, and correct line of best fit
- Part (B) 1 point: Correct answer
- Part (C) 1 point: Correct answer

Conclusion

Overall, the results on the second Junior High Mathematics Program Assessment are disappointing. The results indicate that there is work to be done in reviewing and delivering the curriculum.

There may have been factors that influenced the results. For example, there was no stake set by the department for students who wrote the assessment, and therefore they may not have put forth their best efforts. It is difficult to report with certainty on the extent of all the factors that may have influenced the results; nevertheless, since most students attempted the questions, any inferences are based on the quality or lack thereof in students' responses.

Based on the analyses of student work and results, we are providing suggestions that we hope will lead to significant improvement for student achievement.

For student achievement ...

- Mental math and estimation must be a regular part of the math program. Students need to understand when calculators are appropriate to do computation and when it is best to use mental math strategies.
- Paper-and-pencil computational procedures are best learned by connecting concrete, pictorial, verbal, and contextual representations with symbolic procedures. The failure of many students to be able to use paper-and-pencil procedures might be related to learning that is based on memorization of algorithms, rather than being based on a true understanding.
- Communicating a solution to a problem requires a logical series of steps that permits the reader to fully understand the thinking behind the solution. Students must be discouraged from producing work that is sloppy and disorganized. Furthermore, students must use mathematically correct number sentences. For example, it is mathematically incorrect to write 4+3=7+5=12.
- Number sense includes understanding the relationships among numbers and the meaning of operations and being able to work with numbers flexibly as well as "sensing" the reasonableness of their answers.

Did you know ...

Almost 80 percent of teachers feel the support at the board level for teaching math has been very beneficial.

CONCLUSION

Did you know .

More than 70 percent of teachers believe the provincial Department of Education has provided good support documents.

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- Proportionality and proportional reasoning are major concepts' developed in middle school. Students should solve problems involving proportions using a variety of strategies, which may include scaling or unit-rate. This type of thinking is an essential understanding for solving many real-life problem situations. Students will require these skills in most subject areas, especially the sciences.
- Students should be given opportunities to find patterns and generalize, using representations such as graphs, tables, and equations. These activities are the cornerstones for developing the algebraic thinking needed to succeed in higher mathematics.
- Students need to develop benchmarks and apply strategies to determine measurements and be able to easily convert from one unit to another. They must also be given opportunities to compose and decompose two- and three-dimensional shapes to determine their area and their volume. Students need to distinguish between perimeter and area. They need to have many experiences using measurement tools.
- Students need experience in applying transformations including reflections, translations, rotations, and dilatations on the co-ordinate plane.
- Students must be able to interpret a variety of graphs, including scatter plots and box plots. It is important for students to be complete and accurate when creating graphs by giving the graph a title, using appropriate scales, and labelling axes.
- Students must develop proper use of mathematical language, including multiple meanings of some terms. Writing is a key strategy for improving mathematical understanding and language.

The results indicate that many students are not achieving the outcomes in the prescribed curriculum. All partners in education need to reflect on the results of this assessment and use them to improve student learning.

The greatest challenge for improving achievement in mathematics is to organize and deliver a coherent mathematics program that is focussed on the outcomes listed for each grade level in the provincial curriculum guides. There are many excellent resources available to assist teachers in helping their students learn the mathematical ideas of the Atlantic Canada Mathematics Curriculum. The challenge for all is to maximize the use of these resources in planning, organizing, and delivering a coherent mathematics program.

Notes

CONTINUING & COHERENT MATHEMATICS PROGRAM

A STUDY DOCUMENT FOR EDUCATORS AT THE JUNIOR HIGH LEVEL
